Performance Based Learning and Assessment Task

AFDA Probability & Law of Large Numbers

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
Students will investigate probability and the Law of Large numbers. They will conduct experiments for several different number of trials, record data, and calculate the experimental probability. Then, students will analyze the probabilities as the amount of trials increase in each experiment compared to the theoretical probability. Finally, each group will share their results and the entire class will discuss what happened to the probability as the number of trials increased.

II. UNIT AUTHOR:
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III. COURSE:
Algebra Functions & Data Analysis

IV. CONTENT STRAND:
Data Analysis

V. OBJECTIVES:
AFDA.6 The student will calculate probabilities. Key concepts include (e) Law of Large Numbers

VI. REFERENCE/RESOURCE MATERIALS:
Instruction sheets, calculators, dice, coins, multi-sided die, etc. for experiments.

VII. PRIMARY ASSESSMENT STRATEGIES:
The students will be assessed on the completion of the project as evidence by the worksheet calculations and conclusions written on the sheet.

The assessment list & rubric is attached.

VIII. EVALUATION CRITERIA:
Evaluation Rubric and Benchmarks Attached

IX. INSTRUCTIONAL TIME:
1 class period (Block)/ 1-2 class periods (Traditional)

AFDA Probability & Law of Large Numbers

Strand
Data Analysis

Mathematical Objective(s)
Students will notice that the experimental and theoretical probabilities differ over smaller samples but converge with larger samples.

Related SOL AFDA.6 The student will calculate probabilities. Key concepts include (e) Law of Large Numbers

NCTM Standards

- Understand and apply basic concepts of probability Apply and adapt a variety of appropriate strategies to solve problems
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Develop and evaluate inferences and predictions that are based on data
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):
Using Formulas- Experimental Error

Materials/Resources
- Class set of Graphing Calculators
- Group Instruction and Record Sheet for experiments (Attached)
- Six sided Dice, coins, multi-sided die for groups
- Colored pencils for graphing

Assumption of Prior Knowledge
- Students should know how to calculate basic probability
- Students should know how to plug in numbers into a formula
- Students should know how to conduct experiment
- Students should know how to collect and graph data

Introduction: Setting Up the Mathematical Task

- You will perform an experiment in which you explore the nature of probability in relation to the number of trials. As a group, decide on the random generator you want to use (coin, six sided die, another number of sided die). You will perform your experiment 5 times using 5 different number of trials that are between 1 and 100. After you complete the 5 experiments, you will summarize your findings and discuss with the class. After the class discussion, you will draw your conclusion about the probabilities based on your experiment and the discussion.
- Each group will choose an event to investigate (die, coin, multi-sided die, etc.) Calculate the theoretical probability of each outcome and then choose 5 different trials to consider from 1-100. Students should be working in groups of 4, each with a role for each experiment: Observer, Recorder, Calculator and Conductor- the roles will change with each new experiment set. Conduct the experiment and calculate the experimental probabilities and experimental error for each trial set. Answer the questions as a group together. Then, your group will formulate a conclusion about what the significance of using different number of trials may be.
- 1 class period (Block)/ 1-2 class periods (Traditional)
- In groups of 4, students will conduct experiments and discover what happens to experimental probability.
- The teacher will review how to find probability of an event with class and review the term "Theoretical" and "Experimental" probability in the context of an event.
- In groups, students will have roles to play for each experiment and will change roles with each new task.
- Students will work within their group and then each group will share results at the end.
- The worksheet will move students along the process. (Pages 6-13 given 1st, pg 14-15 at end and used for discussion, class activity, and conclusion, pg 16 used for Individual Closure) Assessment list and Rubric on pages 6-7.
- The teacher will float around and listen for groups making interjections only when needed and asking only facilitating questions.
- Class discussion at the end will allow each group to share their probabilities from the beginning (each being different) with a shared understanding of the Law of Large numbers after discussion.

Student Exploration

Small Group Work (Groups of 4 where possible)
Whole Class Sharing/Discussion (For Closure)

Student/Teacher Actions:

- The assessment list will provide the group with specific items that are necessary to get complete marks for the activity.
- The teacher should be floating to make sure directions are being followed- making sure each student in the group has a role.
- The teacher should have available several ideas and questions for reminding students on how to graph data in different ways.
- The teacher will facilitate a discussion at the end of class, allowing students to discuss what happened as the number of trials increase.
- The class will complete activity with each person rolling 10 trials of a six sided die and combining all of the data together to test what happens with very large number of trials.
- The teacher will define the Law of Large numbers for the class- ask for volunteers to explain how it applies to the class activity, give the conclusion write-up page to each group.
- Students will complete closure slip as individuals.

Monitoring Student Responses

- Students Should:
    - Respectfully communicate with their group
    - Listen to each member's contribution
    - Take their role seriously and change roles with each experiment
    - Communicate their thinking while answering questions as a group

- **Closure Activity (either as exit slip or hw)**
    - Question 1: What is the difference between experimental and theoretical probability?
    - Question 2: What happened to the experimental probability as the number of trials increased?
    - Question 3: Explain the Law of Large Numbers and how today's activity shows its application.

Assessment List and Benchmarks

- Group Assessment List attached
    - Total Points Possible: 20
- Rubric Attached
- Benchmark Attached

Assessment List

No.	Element	Point Value	Self-Assessment	Teacher Assessment
1	Group Assigns Roles for Each Experiment	2		
2	Part 1 calculations for theoretical probability complete	2		
3	Trials are varied from 1 to 100	2		
4	Data recorded and calculations complete	2		
5	Graphs are included for each experiment	2		
6	Experimental Error is calculated for each experiment	2		
7	Summary results completed after 5^{th} Trial to prepare for discussion	2		
8	Conclusion is written in paragraph form	2		
9	Worksheet is turned in on time	2		
10	Worksheet is neat	2		

Total Possible: 20

Rubric

No.	Element	Point Value		
		0	**1**	**2**
1	Group Assigns Roles for Each Experiment	Roles not assigned	Roles not changed	Roles assigned for each exp.
2	Part 1 calculations for theoretical probability complete	Not complete	Somewhat complete	All Complete
3	Trials are varied from 1 to 100	Not varied	Somewhat varied	Each is different
4	Data recorded and calculations complete	Not complete	Somewhat complete	All Complete
5	Graphs are included for each experiment	Graphs not included	Some graphs included	All Graphs included
6	Experimental Error is calculated for each experiment	Not Calculated	Calculated for some	Calculated for All
7	Summary results completed after 5th Trial to prepare for discussion	Not complete	Somewhat complete	Complete
8	Conclusion is written in paragraph form	Conclusion not present	Conclusion present but not written as paragraph	Conclusion included written as paragraph
9	Worksheet is turned in on time	Not turned-in	Not on-time	Turned in on-time
10	Worksheet is neat	Not neat	Somewhat neat	Neat

AFDA Probability & Law of Large Numbers

Task: You will perform an experiment in which you explore the nature of probability in relation to the number of trials. As a group, decide on the random generator you want to use (coin, six sided die, another number of sided die). You will perform your experiment 5 times using 5 different number of trials that are between 1 and 100. After you complete the 5 experiments, you will summarize your findings and discuss with the class. After the class discussion, you will draw your conclusion about the probabilities based on your experiment and the discussion.

Group Members:_____

Each member of the group will be assigned a role for each part of the activity from Conductor (does the experiment), Observer (watches and states outcome), Recorder (notes the outcome in a table), Calculator (calculates probabilities and graphs).

REMEMBER:

Probability: $\frac{\text{\# success (what we want)}}{\text{\# Possible Outcomes}}$

Experimental Error: $\frac{\text{(Experimental probability – Theoretical Probability)}}{\text{Theoretical Probability}} \times 100\ \%$

I. **Theoretical Probability**: List each outcome of the sample space in the table below and calculate the corresponding theoretical probability of each.

List your chosen Random Number Generator:_____

The charts below are listed for convenience. You may not use all of the table blocks below.

Sample Space												
Probability												

**Remember that the sum of the probabilities of the sample space should sum to 1.*

II. **Experimental Probability:** Each group will perform the experiment 5 times using 5 different number of trials between **1 and 100**. The group will draw conclusions about the probabilities in relation to the theoretical probability and discuss with the class at the end of the period.

Conduct Each Experiment with a chosen number of trials between 1 and 100 and record your data for the trials, graph the frequency of your data outcomes, calculate probabilities, and then calculate experimental error of the first sample after each section.

a. _____ Trials:

Group Roles

Conductor: _____ Observer: _____

Recorder: _____ Calculator: _____

FREQUENCY:

PROBABILITY:

GRAPH:

Calculate the Experimental Error for **the first sample** outcome versus the theoretical probability.

b. _____ Trials:

Group Roles

Conductor: _____ Observer: _____

Recorder: _____ Calculator: _____

FREQUENCY:

PROBABILITY:

GRAPH:

Calculate the Experimental Error for **the first sample** outcome versus the theoretical probability.

c. _____ Trials:

Group Roles

Conductor: _____ Observer: _____

Recorder: _____ Calculator: _____

FREQUENCY:

PROBABILITY:

GRAPH:

Calculate the Experimental Error for **the first sample** outcome versus the theoretical probability.

d. _____ Trials:

Group Roles

Conductor: _____ Observer: _____

Recorder: _____ Calculator: _____

FREQUENCY:

PROBABILITY:

GRAPH:

Calculate the Experimental Error for **the first sample** outcome versus the theoretical probability.

e. _____ Trials:

Group Roles

Conductor: _____ Observer: _____

Recorder: _____ Calculator: _____

FREQUENCY:

PROBABILITY:

GRAPH:

Calculate the Experimental Error for **the first sample** outcome versus the theoretical probability.

III. **Summary:** Summarize your results about the theoretical and experimental probabilities in relation to the number of trials used for each experiment. Be prepared to speak about your summary during the class discussion.

Let's put it to the test!

IV. Each person conduct 10 trials (samples) for a six sided die

USE THE CLASS DATA TO FILL OUT THE FREQUENCY TABLE, GRAPH THE FREQUENCY AND CALCULATE PROBABILITIES

_____ TRIALS

TOTAL # FROM CLASS

	Frequency
1	
2	
3	
4	
5	
6	

#	Probability
1	
2	
3	
4	
5	
6	

GRAPH:

Experimental Error for 1:

Discussion Questions:

What happens to the probability as the trial numbers increase?

What happens to the Experimental error as the trial numbers increase?

V. The ***Law of Large Numbers*** is a principle of probability according to which the frequencies of events with the same likelihood of occurrence even out, given enough trials or instances. As the number of experiments increases, the actual ratio of outcomes will converge on the theoretical, or expected, ratio of outcomes.

Conclusion: Write a paragraph outlining your conclusions from today's activity. Include your thoughts to the following questions.

- What is your group's conclusion about what happens when the number of trials increases?
- Why do you think it is important to do multiple trials (samples) of an experiment?
- How can the principle of the Law of Large Numbers be applied in "the real world", outside of this experiment? (hint: think of times when experiments need to be conducted for reliability)

INDIVIDUAL CLOSURE:

Name _____

Each Student should complete questions and attach to group work.

- Question 1: What is the difference between experimental and theoretical probability?

- Question 2: What happened to the experimental probability as the number of trials increased?

- Question 3: Explain the Law of Large Numbers and how the activity shows its application.

BENCHMARK: *This benchmark shows different types of charts, probability representations and sample answers. This is not an answer key.*

AFDA Probability & Law of Large Numbers

Task: You will perform an experiment in which you explore the nature of probability in relation to the number of trials. As a group, decide on the random generator you want to use (coin, six sided die, another number of sided die). You will perform your experiment 5 times using 5 different number of trials that are between 1 and 100. After you complete the 5 experiments, you will summarize your findings and discuss with the class. After the class discussion, you will draw your conclusion about the probabilities based on your experiment and the discussion.

Group Members:___Cris, Yvonne, Joni, Grace_____

Each member of the group will be assigned a role for each part of the activity from Conductor (does the experiment), Observer (watches and states outcome), Recorder (notes the outcome in a table), Calculator (calculates probabilities and graphs).

REMEMBER:

Probability: $\frac{\text{\# success (what we want)}}{\text{\# Possible Outcomes}}$

Experimental Error: $\frac{\text{(Experimental probability – Theoretical Probability)}}{\text{Theoretical Probability}} \times 100$ **%**

I. Theoretical Probability: List each outcome of the sample space in the table below and calculate the corresponding theoretical probability of each.

List your chosen Random Number Generator:___6-sided die_____

The charts below are listed for convenience. You may not use all of the table blocks below.

Sample Space	1	2	3	4	5	6						
Probability	1/6	1/6	1/6	1/6	1/6	1/6						

**Remember that the sum of the probabilities of the sample space should sum to 1.*

II. Experimental Probability: Each group will perform the experiment 5 times using 5 different number of trials between **1 and 100**. The group will draw conclusions about the probabilities in relation to the theoretical probability and discuss with the class at the end of the period.

Used a 6-sided Die

a. 10 Trials:

Group Roles

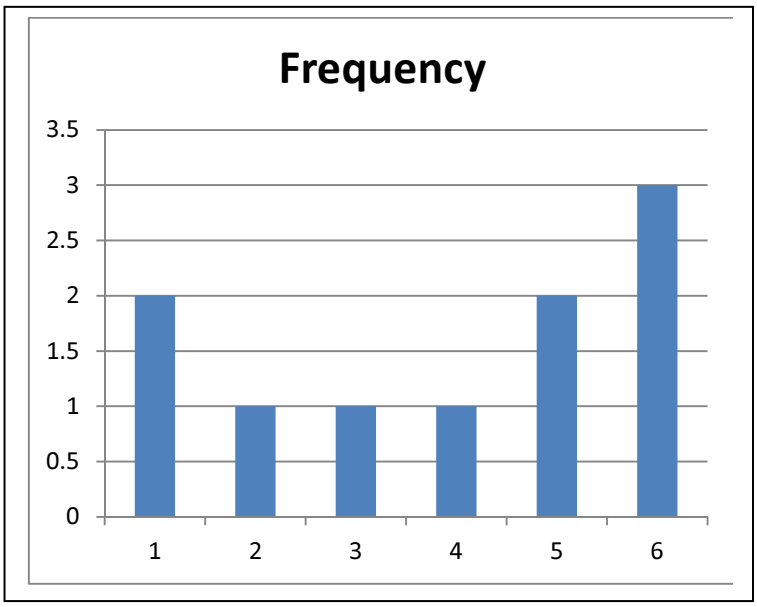
Conductor: ___Cris_____ Observer: _____Yvonne_____

Recorder: _____Joni_____ Calculator: _____Grace_____

#	Frequency
1	2
2	1
3	1
4	1
5	2
6	3

#	Probability
1	2/10
2	1/10
3	1/10
4	1/10
5	2/10
6	3/10

GRAPH:



Calculate the Experimental Error for 1. (2/10 – 1/6) / 1/6 = 20%

b. 20 Trials:

Group Roles

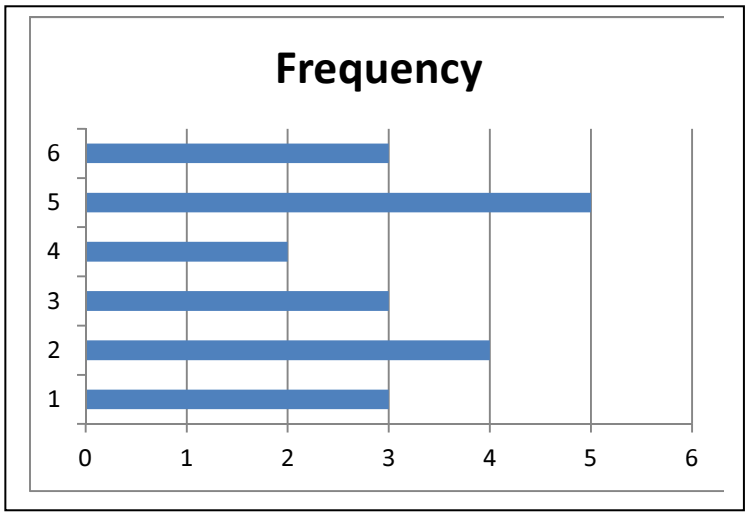
Conductor: __Grace_____ Observer: _____Cris_____

Recorder: _Yvonne_____ Calculator: ___Joni_____

	Frequency
1	3
2	4
3	3
4	2
5	5
6	3

#	Probability
1	3/20
2	4/20
3	3/20
4	2/20
5	5/20
6	3/20

GRAPH:



Calculate the Experimental Error for 1: -10%

c. 50 Trials:

Group Roles

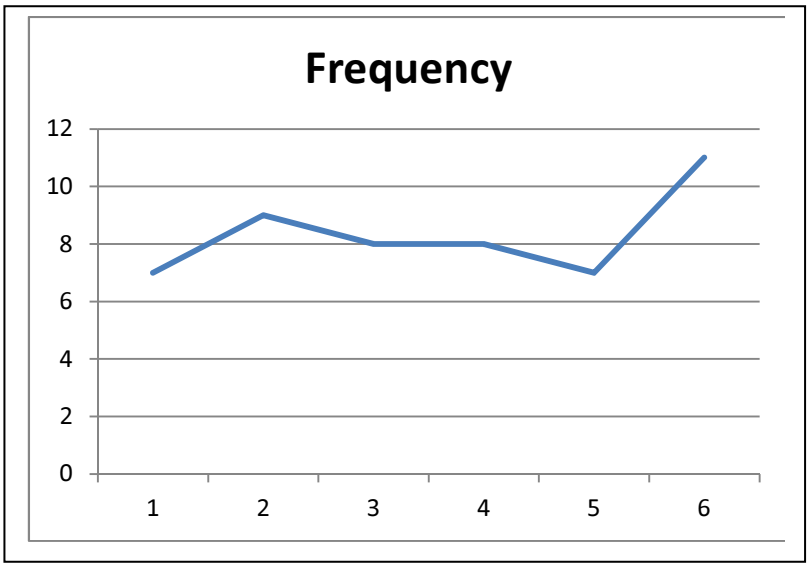
Conductor: _____Joni_____ Observer: ___Grace_____

Recorder: _____Cris_____ Calculator: __Yvonne_____

	Frequency
1	7
2	9
3	8
4	8
5	7
6	11

#	Probability
1	7/50
2	9/50
3	8/50
4	8/50
5	7/50
6	11/50

GRAPH:



Calculate the Experimental Error for 1: -4%

d. 100 Trials:

Group Roles

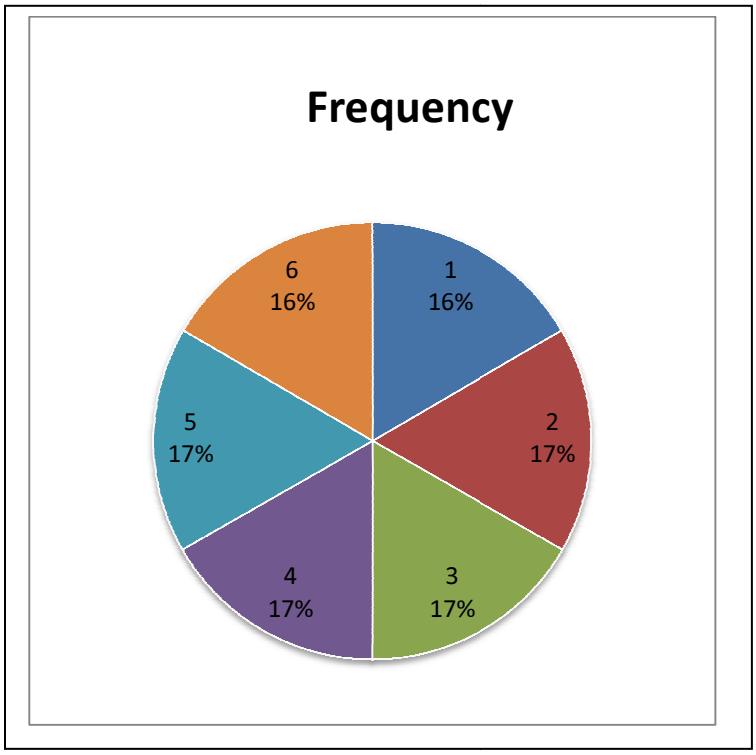
Conductor: ____Yvonne_____ Observer: _____Joni_____

Recorder: _____Grace_____ Calculator: ___Cris_____

	Frequency
1	16
2	16
3	17
4	16
5	18
6	17

#	Probability
1	.16
2	.16
3	.17
4	.16
5	.18
6	.17

GRAPH:



Calculate the Experimental Error for 1: -4%

III. Summary: As the number of trials increase, the experimental probability gets closer to theoretical probability.

Let's put it to the test!

IV. Each person conduct 10 trials (samples) for a six sided die

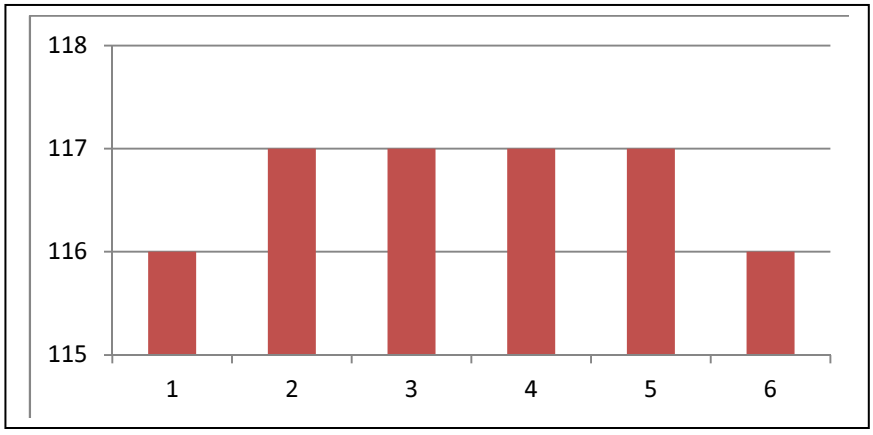
USE THE CLASS DATA TO FILL OUT THE FREQUENCY TABLE, GRAPH THE FREQUENCY AND CALCULATE PROBABILITIES

____700____ TRIALS
TOTAL # FROM CLASS

	Frequency
1	116
2	117
3	117
4	117
5	117
6	116

#	Probability
1	.1657
2	.1671
3	.1671
4	.1671
5	.1671
6	.1657

GRAPH:



Experimental Error for 1: -0.6%

QUESTIONS:

What happens to the probability as the trial numbers increase?
The probabilities get closer to the same number.

What happens to the Experimental error as the trial numbers increase?
The error decreases as the number of trials increases.

The ***Law of Large Numbers*** is a principle of probability according to which the frequencies of events with the same likelihood of occurrence even out, given enough trials or instances. As the number of experiments increases, the actual ratio of outcomes will converge on the theoretical, or expected, ratio of outcomes.

V. Conclusion: Write a paragraph outlining your conclusions from today's activity. Include your thoughts to the following questions.

- What is your group's conclusion about what happens when the number of trials increases?
- Why do you think it is important to do multiple trials of an experiment?
- How can the principle of the Law of Large Numbers be applied in "the real world", outside of this experiment? (hint: think of times when experiments need to be conducted for reliability)

As the number of trials increase, the probability of the experiment gets closer to the theoretical probability. It is important to do multiple trials in order to get closer the actual probability and get a better feel for the true probability. The Law of Large Numbers suggests that as we increase trials the actual ratio gets close to expected values. We can apply this to scientists doing experiments today. We would want them to do many trials on things such as medicine in order to get true results.